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THE INITIAL BOUNDARY VALUE PROBLEM OF GUN DYNAMICS SOLVED BY FI--ETC(U)
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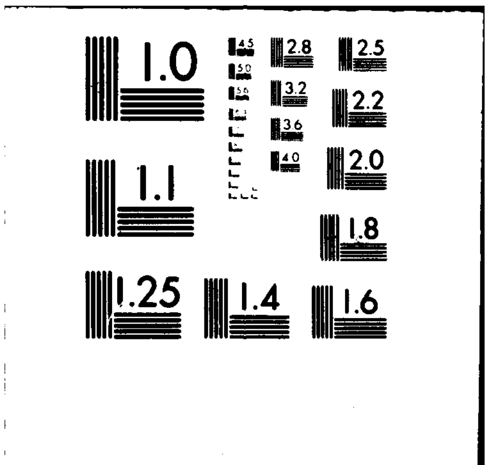

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THE INITIAL BOUNDARY VALUE PROBLEM OF GUN DYNAMICS SOLVED BY FINITE ELEMENT-UNCONSTRAINED VARIATIONAL FORMULATIONS (U)

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1. INTRODUCTION

The purpose of this paper is to introduce an efficient method, which is quite general and easy to use, to the solution of the gun dynamic problems, to describe some of the features associated with a finite element computer program, and to present some initial results.

The basic concept of unconstrained, adjoint variational formulation for linear problems was described in an earlier paper [1]. Its advantage over constrained methods in obtaining approximate solutions has been demonstrated for both conservative (self-adjoint) and unconservative (nonself-adjoint) problems [2]. In comparison with Galerkin procedure, the unconstrained, adjoint variational formulation has a further advantage in the freedom of selecting shape functions which have less requirement on differentiability and which are not required to satisfy any of the end conditions. The same concept was extended to solution formulation of initial value problems [3]. In view of the generality of this approach and its easy adaptability to finite element discretizations, it appears to be quite attractive in seeking solutions to the complicated problems associated with the dynamics of gun systems.

The basis of the present formulation for more general cases has been given in a previous paper [4]. The special problem of a uniform gun tube is treated here for demonstrated purposes. The differential equation and initial boundary conditions are given in Section 2. An unconstrained variational statement which is equivalent to the given governing equations is stated in Section 3. The details of some of the

special features on finite element discretization are described in Section 4. Finally some preliminary results are presented in Section 5.

2. GOVERNING EQUATIONS

The motion of a gun tube modeled by the lateral deflection of an Euler-Bernoulli beam is shown in Fig. 1. The differential equation in nondimensional form is

$$y'''' + (-\bar{P} + g \sin \alpha)[(1-x)y']' + \gamma^2 \ddot{y}$$
$$= -\bar{P}y''H(\bar{x}-x) - \gamma^2 m[\beta^2 t^2 y'' + 2\beta t \dot{y}' + \ddot{y}]\bar{\delta}(\bar{x}-x)$$
$$-(gm \cos \alpha)\bar{\delta}(\bar{x}-x) - g \cos \alpha \qquad (1)$$

where

$\bar{P} = \pi R^2 p$
$y = y(x,t)$, the tube deflection
x = spatial axis along the tube's length, $0 \leq x \leq 1$
t = time axis, $0 \leq t \leq T$, T is the time limit of interest
α = elevation angle
m = projectile mass
β = acceleration of the projectile, assumed to be constant
p = bore pressure, assumed to be constant
g = gravitational acceleration
$\bar{x} = (1/2)\beta t^2$, projectile position
$H(x)$ = the Heavenside step function
$\bar{\delta}(x)$ = the Dirac delta function
R = tube inner radius
$\gamma = c/T$

The constant c is defined by $(\rho A \ell^4/(EI))^{1/2}$ where ρ, E are density, Young's modulus of the tube material; ℓ, A, I, the length, area, and second moment of cross-section of tube, respectively (see Ref. [5]). Also in Eq. (1), a prime (') denotes a differentiation with respect to x and dot (·), a differentiation with respect to t. The derivation of this equation and the end conditions which follow have been previously given [4,5] and will not be repeated here.

The initial condition, or more appropriately, the end conditions in time are

$$\dot{y}(x,0) = 0 \ ; \quad \dot{y}(x,1)[1 + m\bar{\delta}(\tfrac{1}{2}\beta - x)] + k_7[y(x,0) - Y(x)] = 0 \qquad (2)$$

(442)

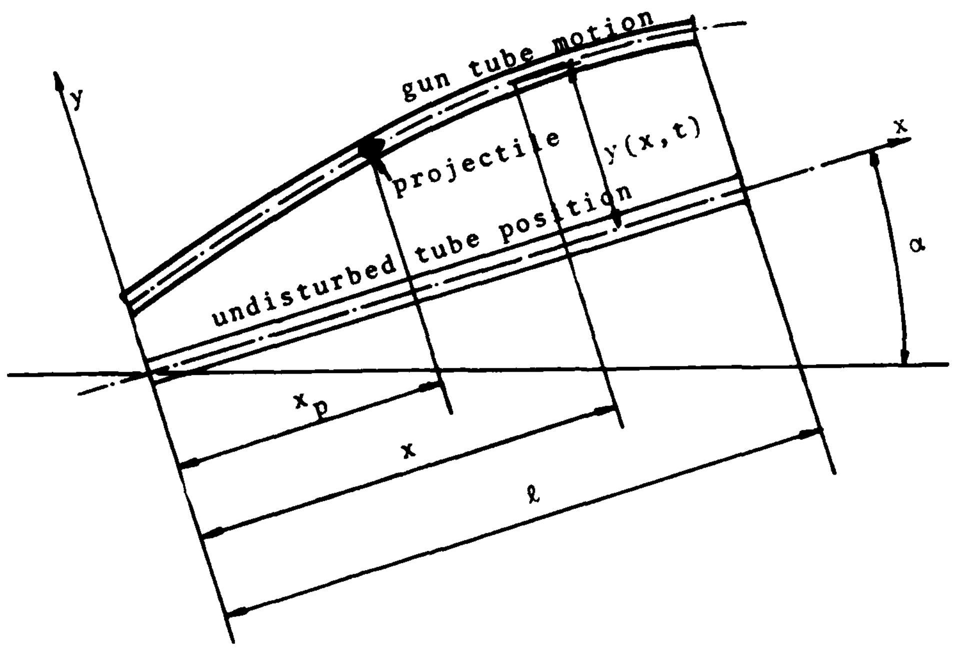

Fig. 1 A schematic drawing of the problem configuration

where the constant k_7 is introduced in conjunction with the unconstrained variational formulation (Section 3) so that if one takes k_7 to be infinite, the initial displacement $y(x,0)$ is forced to be identical to the prescribed shape $Y(x)$. Based on similar reasonings the boundary conditions have been shown to be the following:

$$y''(0,t) - k_2 y'(0,t) = 0 \tag{3a}$$

$$y''(1,t) + k_4 y'(1,t) = 0 \tag{3b}$$

$$y'''(0,t) + k_1 y(0,t) + (-\bar{P} + g\cos\ )y'(0,t)$$
$$+ \bar{P}y'(0,t)H(\tfrac{1}{2}\beta t^2) + m\beta^2 y'(0,t)\bar{\delta}(\tfrac{1}{2}\beta t^2) = 0 \tag{3c}$$

and

$$y'''(1,t) - k_3(1,t) + \bar{P}y'(1,t)H(\tfrac{1}{2}\beta t^2 - 1) + m\beta^2 y'(1,t)\bar{\delta}(\tfrac{1}{2}\beta t^2 - 1) = 0 \tag{3d}$$

where k_i, $i = 1, 2, 3, 4$, are the appropriate elastic spring constants at the supports.

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3. UNCONSTRAINED VARIATIONAL STATEMENT

Through integrations-by-parts, it is straight forward to show that the following variational statement is equivalent to the differential equation and end conditions stated in Section 2.

$$\delta I = (\delta I)_y = \sum_{i=1}^{12} (\delta I_i)_y - \sum_{j=1}^{3} (\delta J_j) = 0 \tag{4}$$

with

$$I_1=\int_0^1 \int_0^1 y''y^{*''}dxdt \; ; \; I_2=(\bar{P}-g \sin \alpha)\int_0^1 \int_0^1 (1-x)y'y^{*'}dxdt$$

$$I_3=-\gamma^2\int_0^1 \int_0^1 \ddot{y}y^*dxdt \; ; \; I_4=-\bar{P}\int_0^1 \int_0^1 y'y^{*'}H(\bar{x}-x)dxdt$$

$$I_5=-\bar{P}\int_0^1 \int_0^1 y'y^{*'}\bar{\delta}(\bar{x}-x)dxdt \; ; \; I_6=-m\beta^2\gamma^2\int_0^1 \int_0^1 t^2y'y^{*'}\bar{\delta}(\bar{x}-x)dxdt$$

$$I_7=-m\beta^2\gamma^2\int_0^1 \int_0^1 ty'y^{*'}\bar{\delta}'(\bar{x}-x)dxdt \; ; \; I_8=2m\beta\gamma^2\int_0^1 \int_0^1 ty'y^*\bar{\delta}(\bar{x}-x)dxdt \tag{5}$$

$$I_9=-m\gamma^2\int_0^1 \int_0^1 \ddot{y}y^*\bar{\delta}(\bar{x}-x)dxdt \; ; \; I_{10}=-m\gamma^2\int_0^1 \int_0^1 \dot{y}y^*\bar{\delta}(\bar{x}-x)dxdt$$

$$I_{11}=\int_0^1 \{k_1y(0,t)y^*(0,t) + k_2y'(0,t)y^*(0,t)$$

$$+ k_3y(1,t)y^*(1,t) + k_4y'(1,t)y^{*'}(1,t)dt$$

$$I_{12}=k_7\int_0^1 y(x,0)y^*(x,1)dx$$

and

$$J_1 = -g \cos \alpha\int_0^1 \int_0^1 y^*dxdt$$

$$J_2 = -gm \cos \alpha\int_0^1 \int_0^1 y^*\bar{\delta}(\bar{x}-x)dxdt \tag{6}$$

$$J_3 = k_7\int_0^1 Y(x)y^*(x,1)dx$$

(444)

The variational statement of Eq. (4) will serve as the basis of our finite element solutions.

4. FINITE ELEMENT DISCRETIZATION

The purpose of the discretization is to enable one to write the variational statement of Eq. (4), which is a functional of continuous functions y and y*, etc., in the form of a matrix equation

$$\delta \underset{\sim}{Y}^{*T} \underset{\sim}{K}\, \underset{\sim}{Y} = \delta \underset{\sim}{Y}^{*T} \underset{\sim}{F} \tag{7}$$

where $\underset{\sim}{Y}$, $\underset{\sim}{Y}^*$ are the "global" generalized coordinates vectors. $\underset{\sim}{K}$ is the "stiffness" matrix, and $\underset{\sim}{F}$ the "force" vector. These terminology are patented after the static structural analysis, but they do not necessarily have the physical meanings of those adjectives attached to them. Since the variational statement associated with Eq. (7) is unconstrained, the equation leads directly to

$$\underset{\sim}{K}\, \underset{\sim}{Y} = \underset{\sim}{F} \tag{8}$$

which can be solved for $\underset{\sim}{Y}$ if $\underset{\sim}{K}$ and $\underset{\sim}{F}$ are properly defined. The process by which $\underset{\sim}{K}$ and $\underset{\sim}{F}$ are assembled and the relation between $\underset{\sim}{Y}$ and the desired solution y(x,t) will be described here in this section.

The first step is to write down the expressions in the variational statement in terms of the element variables. A grid scheme of elements is shown in Fig. 2. In this figure, the nondimensional length of the gun tube is divided into K equal segments and the time range of interest into L equal segments. The result is then a set of K x L rectangular elements. In the equations that follow n, the sub- or super-scripts m,n denote the association with the m^{th},n^{th} segments or the $(m,n)^{th}$ element. Define the relation between the local coordinates (ξ,η) of the $(m,n)^{th}$ element and the global coordinates (x,t) by

$$\xi = \xi^{(m)} = Kx - m + 1$$

$$\eta = \eta^{(n)} = Lt - n + 1 \tag{9}$$

Or,

$$x = \frac{1}{K}(\xi+m-1) \quad ; \quad t = \frac{1}{L}(\eta+n-1)$$

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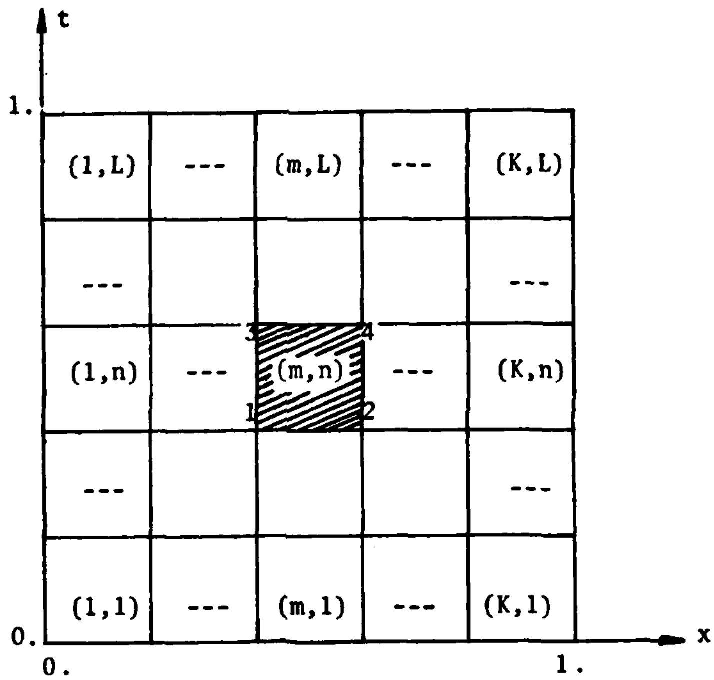

Fig. 2 Finite element grid scheme in space- and time-coordinates. Also shown: node point ordering in a typical element (m,n).

One then can write from Eqs. (5) and (6)

$$\delta I_1 = \sum_{m=1}^{K} \sum_{n=1}^{L} \frac{K^3}{L} \int_0^1 \int_0^1 y''_{(m,n)} \delta y^{*''}_{(m,n)} d\xi d\eta$$

$$\delta I_2 = \sum\sum \frac{1}{L} (P-g \sin \alpha) \int_0^1 \int_0^1 [(K-m+1) - \xi] y'_{(m,n)} \delta y^{*'}_{(m,n)} d\xi d\eta$$

$$\delta I_3 = - \sum\sum \frac{L}{TK} \int_0^1 \int_0^1 \dot{y}_{(m,n)} \delta \dot{y}^{*}_{(m,n)} d\xi d\eta$$

$$\delta I_4 = - \sum\sum \frac{\bar{P}K}{L} \int_0^1 \int_0^1 y'_{(m,n)} \delta y^{*'}_{(m,n)} H_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta$$

$$\delta I_5 = \sum\sum \frac{\bar{P}K}{L} \int_0^1 \int_0^1 y'_{(m,n)} \delta y^{*}_{(m,n)} \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta \qquad (10a)$$

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$$\delta I_6 = -\sum\sum \frac{m\beta^2 K\gamma^2}{L^3} \int_0^1 \int_0^1 [(m-1)^2+2(m-1)\eta+\eta^2] y'_{(m,n)} \delta y^{*\prime}_{(m,n)} \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta$$

$$\delta I_7 = -\sum\sum \frac{m\beta^2 K\gamma^2}{L^3} \int_0^1 \int_0^1 [(m-1)^2+2(m-1)\eta+\eta^2] y'_{(m,n)} \delta y^{*\prime}_{(m,n)} \bar{\delta}'_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta$$

$$\delta I_8 = \sum\sum \frac{2m\beta\gamma^2}{L} \int_0^1 \int_0^1 [(m-1)+\eta] \dot{y}'_{(m,n)} \delta y^*_{(m,n)} \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta$$

$$\delta I_9 = -\sum\sum \frac{mL\gamma^2}{K} \int_0^1 \int_0^1 \dot{y}_{(m,n)} \delta \dot{y}^*_{(m,n)} \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta$$

$$\delta I_{10} = -\sum\sum \frac{mL\gamma^2}{K} \int_0^1 \int_0^1 \dot{y}_{(m,n)} \delta y^*_{(m,n)} \dot{\bar{\delta}}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta$$

$$\delta I_{11} = \sum_{n=1}^{L} \frac{1}{LK} \int_0^1 [k_1 y_{(1,n)} \delta y^*_{(1,n)} + k_2 K y'_{(1,n)} \delta y^{*\prime}_{(1,n)} + k_3 y_{(K,n)} \delta y^*_{(K,n)} + k_4 K y'_{(K,n)} \delta y^{*\prime}_{(K,n)}] d\eta$$

$$\delta I_{12} = \sum_{m=1}^{K} \frac{k_7}{K} \int_0^1 y_{(m,1)} \delta y^*_{(m,L)} d\xi \qquad (10b)$$

and

$$\delta J_1 = -\sum_{m=1}^{K} \sum_{n=1}^{L} \frac{\gamma^2 g \cos\alpha}{KL} \int_0^1 \int_0^1 \delta y^*_{(m,n)} d\xi d\eta$$

$$\delta J_2 = -\sum\sum \frac{\gamma^2 gm \cos\alpha}{L} \int_0^1 \int_0^1 \delta y^*_{(m,n)} \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta \qquad (11)$$

$$\delta J_3 = \sum_{m=1}^{K} \frac{k_7}{K} \int_0^1 Y_{(m)}(\xi) \delta y^*_{(m,L)} d\xi$$

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Now, the shape function vector $\underset{\sim}{a}(\xi,\eta)$ is introduced

$$y_{(m,n)}(\xi,\eta) = \underset{\sim}{a}^T(\xi,\eta)\underset{\sim}{Y}^{(m,n)}$$

$$y^*_{(m,n)}(\xi,\eta) = a^T(\xi,\eta)Y^{*(m,n)} \tag{12}$$

where $Y(m,n)$ and $Y^*(m,n)$ are the generalized coordinates for the originaladjoint field variables. Thus, equations in the form of Eq. (7) and (8) are obtained in the usual manner of finite element formulation (see, for example, Ref. [5]).

5. NUMERICAL RESULTS

In actual computation, a simplified model of a M68-105 mm cannon tube is used. The source of data can be found in Reference [4]. For the cannon tube, we take

$$E = 3 \times 10^7 \text{ psi} \quad ; \quad \rho = 0.283 \text{ lb/in.}^3$$

$$\ell = 210 \text{ in.} \quad ; \quad A = 28.2 \text{ in.}^2 \quad ; \quad I = 123.6 \text{ in.}^4$$

and, for the projectile,

$$m_p = 20 \text{ lb} \quad ; \quad p \text{ (inbore pressure)} = 40{,}000 \text{ psi}$$

$$\beta \text{ (projectile acceleration)} = 6{,}000{,}000 \text{ in./sec}^2$$

$$v_{average} = 25{,}000 \text{ ft/sec} \quad ; \quad T = 0.008 \text{ sec}$$

Consequently,

$$c = (\rho A \ell^4/(EI))^{1/2} = 0.104 \text{ sec}$$

$$\ell/c = 2100 \text{ in./sec} \quad ; \quad \gamma = c/T = 13.00$$

In the finite element discretization, we have chosen a grid scheme of 5 x 5 = 25 elements as shown in Fig. 2. The shape functions used are cubic in x and linear in t. As an illustrative example, the support conditions are those of a cantilever. Thus k_1 and k_2 are infinite. In actual computations, they are taken to be very large numbers compared with unity (e.g., 10^8). The initial conditions are such that the tube has zero deflection and zero velocity at t = 0.

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TABLE 1. DEFLECTION OF A GUN TUBE $[y(x,t)/\ell] \times 10^4$ AS THE PROJECTILE MOVES THROUGH THE TUBE TILL EJECTION

t/T \ x/ℓ	0	0.2	0.4	0.6	0.8	1.0
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.25	0.0000	0.0151	0.0192	0.0182	0.0184	0.0281
0.50	0.0000	0.0413	0.0786	0.0735	0.0783	0.0986
0.75	0.0000	0.0724	0.1527	0.1768	0.1830	0.2156
1.00	0.0000	0.0905	0.2386	0.3245	0.3411	0.3362

TABLE 2. SLOPE OF A GUN TUBE $y'(x,t) \times 10^4$ AS THE PROJECTILE MOVES THROUGH THE TUBE TILL EJECTION

t/T \ x/ℓ	0	0.2	0.4	0.6	0.8	1.0
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.25	0.0000	0.0511	-0.0079	-0.0121	0.0081	0.0710
0.50	0.0000	0.3132	0.0556	-0.0171	0.1021	-0.0936
0.75	0.0000	0.3758	0.2248	-0.0327	-0.0111	0.2647
1.00	0.0000	1.1419	0.7929	0.3932	0.3423	-0.2530

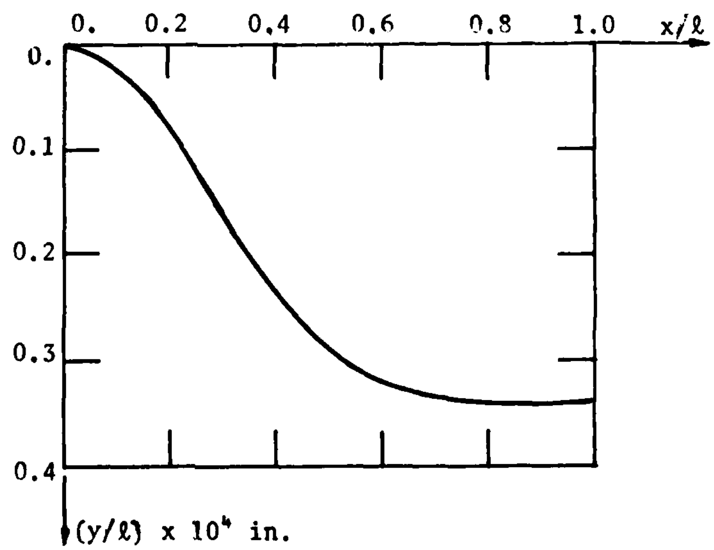

Fig. 3 Deflection of a gun tube at shot ejection.

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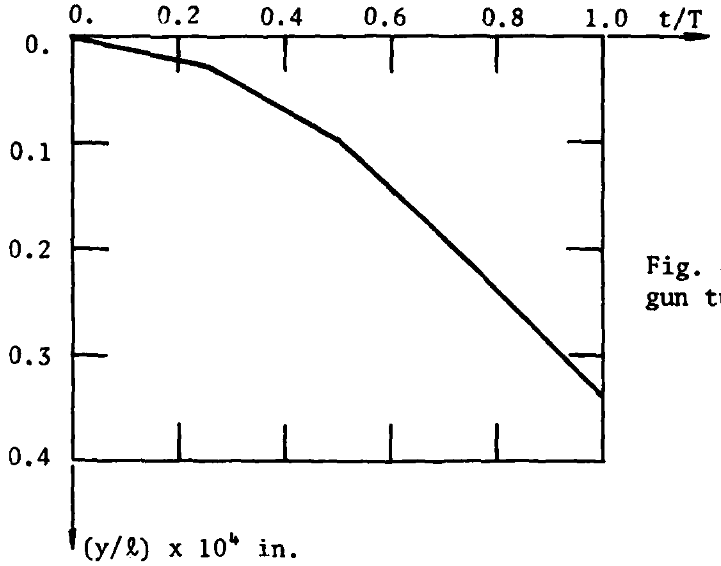

Fig. 4 Muzzle motion of a gun tube during firing.

Tables 1 and 2 contain the results of tube deflection and slope as the projectile moves down the tube from the breech end to the muzzle. For example, at $t = 0.5T$, the deflection represents tube deflection when the projectile is at a location of $x = 0.25\ell$ from the breech end.

Figure 3 depicts the deflection curve of the gun tube at projectile ejection and Figure 4 illustrates the muzzle motion as the projectile moves down the tube.

From Tables 1 and 2 it is observed that the initial data have been recovered for this present unconstrained variational formulation. The data presented here are of the same order of magnitude compared with other analysts [4]. Further verification of the computational results and a parametric study of the relative importance of the various forces involved in the problem will follow in the immediate future.

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